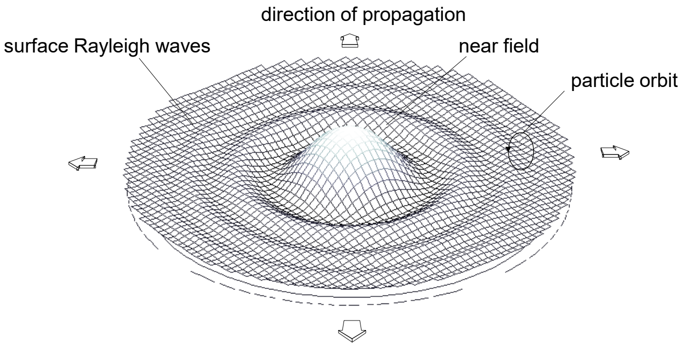
Doctoral Course

Academic Year 2025-26

Propagation of Mechanical Waves in Deformable Solids



PhD Programs: - *Design, Modeling and Simulation in Engineering*
- *Earth and Environmental Sciences*
- *Understanding and Managing Extremes (IUSS Pavia)*

Institutions: University of Pavia and IUSS Pavia
Term: Academic Year 2025-26 – Second Semester

Instructor: Prof. Carlo G. Lai (E-mail: carlogiovanni.lai@unipv.it)
Credits (CFU): 6

Delivery mode: The course is delivered through standard frontal classes. If required by the health situation classes will be delivered live via the web using Zoom

Class duration: 15th January – 6th March 2026 (*32* hours)
Class schedule: Thursday and Friday 4–6 PM. Classroom: F2 @ Polo Cravino (UniPV)

Office hours: Contact the instructor via email.

DESCRIPTION

Scope of the course is to provide in-depth theoretical knowledge and innovative methodological approaches in the study of the phenomena associated with the propagation of mechanical waves in deformable solids. The course is addressed to scholars working in different, yet interacting research fields including structural mechanics, geophysics, seismology, and geotechnical engineering. Thus, it is a cross-domain course intended to bridge the gaps among the above disciplines when studying the propagation of mechanical disturbances in natural (e.g., soils, rocks) and construction materials (e.g., steel, concrete). Although the emphasis of the course is on mathematical and constitutive modeling, an effort is made by the instructor to illustrate applications of the theories to the solution of problems taken from real-world situations. It is expected that at the end of the course the attendees will have mastered the tools of analysis introduced to them so to independently investigate their research topics.

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COURSE OUTLINE

The course begins with a review of a few mathematical concepts including Fourier and Laplace transforms, Bessel functions, spherical harmonics. Afterwards, the two main classes of wave motion represented by hyperbolic and dispersive waves are introduced as they constitute the theoretical framework for the remaining of the course.
Next subject to be discussed is wave propagation in elastic waveguides. Problems examined include vibrations of strings, flexural waves in beams and in thin membranes. A selected number of both free and forced vibration problems will be analysed with reference to infinite and finite 1D and 2D structural members.
The second part of the course focuses on problems of wave propagation in unbounded continua and half-spaces. Topics include wave motions with polar and axial symmetry, propagation of waves in non-homogeneous media, surface Love and Rayleigh waves, the solution of the Lamb problem including a discussion of the differences between 2D versus 3D radiation (Huyghens' principle). The dispersive properties of surface waves will be illustrated with examples drawn from geophysics and seismology in the solution of relevant forward and inverse problems.
The following subject to be studied is wave propagation in linear dissipative continua which includes Boltzmann equations, the elastic-viscoelastic correspondence principle for time-invariant boundary conditions and a discussion on the implications of the principle of physical causality. In connection to the theory of viscoelasticity ideas from fractional calculus will also be introduced.
The course ends with the analysis of moving loads applied at the free-surface of an elastic half-space under subcritical and supercritical regimes. The subject is closely related to the study of the vibrational impact induced by fast and super-fast trains. Time permitting and if of interest to the participants the course will also provide a brief introduction to wave propagation in saturated poroelastic media (Biot's theory).

COURSE REQUIREMENTS

Basic knowledge of Calculus and Mechanics of Deformable Body

COURSE NOTES

The course material is posted at the KIRO E-learning platform accessible @ https://elearning-excu.unipv.it/

REFERENCES

- Achenbach, J.D. (1984). Wave Propagation in Elastic Solids, North-Holland, 425 pp.
- Graff, K.F. (1991). Wave Motion in Elastic Solids, Dover Publications, 649 pp.
- Kausel, E. (2006). Fundamental Solutions in Elastodynamics, Cambridge Press, 251 pp.
- Aki, K. & Richards P.G. (2002). Quantitative Seismology, University Science, 700 pp.
- Verruijt, A. (2010). An Introduction to Soil Dynamics. Springer, New York, 431 pp.
- Lecture notes, scientific articles will be provided throughout the course

ASSESSMENT

An assignment will be handed over and graded at the end of the course.

SCHEDULE OF LECTURES	HOURS	# LECTURES
1. Review of Fourier and Laplace transforms. Special functions	6	1–3
2. Classification of wave motion: hyperbolic and dispersive waves	2	4
3. Waves in elastic waveguides: vibrations of strings, beams and membranes	6	5–7
4. Wave propagation in unbounded media and half-spaces. Surface waves	8	8–11
5. Wave motion with polar and axial symmetry. The cavity source problem	2	12
6. Wave motion in linear dissipative continua. Physical causality	4	13–14
7. Elastic vibrations induced by moving loads: fast and super-fast trains	2	15
8. Poroelasticity and Biot theory. Waves in saturated porous materials	2	16
TOTAL	**32**	